ON THE LIFTING FORCE OF SOURCE AND DIPOLE IN BOUNDED FLUID STREAM

(O PODEMNOI SILE ISTOCHNIKA I DIPOLIA V OGRANICHENNOM POTOKE ZHIDKOSTI)

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This paper presents the values of the lifting force (and of the force of the lateral resistance) acting on hydromechanical singularities as they move at velocities both sufficiently small and large under the free surface of an ideal fluid bounded by walls.

1. We make the usual assumptions of small amplitude wave theory. Let the coordinates x and y be in the free surface of the fluid in its stagnant state, and the z-axis be directed upward. Consider the coordinate system to be permanently connected with the singularity (source, dipole) moving uniformly with velocity v in the direction of the x-axis under the surface of the fluid. The source (dipole) is placed at the point $(0, 0, \zeta)$. The depth of the fluid is denoted by h.

The formulas for the determination of the hydromechanical lifting force A_d, experienced by the source and the dipole, may be written in the following form [1]:

for the source

$$A_d = -\frac{\rho Q^2}{4\pi}\left[\frac{1}{4(\zeta+h)^2} - \frac{1}{\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} d\theta \int_0^\infty \chi(\lambda, \theta)\,\lambda\, d\lambda\right] \tag{1.1}$$

for the dipole

$$A_d = -\frac{\rho m^2}{4\pi}\left[\frac{3}{16(\zeta+h)^4} - \frac{1}{\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \cos^2\theta\, d\theta \int_0^\infty \chi(\lambda, \theta)\,\lambda^3 d\lambda\right] \tag{1.2}$$

Here and subsequently ρ is the mass density of the fluid, g is acceleration due to gravity, Q is the intensity of the source and m is the dipole moment in the direction of the x-axis.

$$\chi(\lambda, \theta) = \frac{e^{-\lambda h}(\nu + \lambda\cos^2\theta)\operatorname{sh} 2\lambda(\zeta + h)}{(\nu \operatorname{th}\lambda h - \lambda\cos^2\theta)\operatorname{ch}\lambda h} \qquad \left(\nu = \frac{g}{v^2}\right) \tag{1.3}$$

In formulas (1.1) and (1.2) the integral sign is understood to designate the principal value of the integral in the Cauchy sense.

If in (1.2) the expression $2\pi v r^3$ is substituted for m, we will then obtain an approximate formula for the hydromechanical lifting force, applied to the sphere of radius r, the center of which is situated at the depth ζ under the free surface of the fluid. The total lifting force A, acting on the sphere, is equal to:

$$A = \frac{4}{3}\pi\rho g r^3 + A_d \tag{1.4}$$

2. Let us find an approximate expression for A_d for sufficiently small values of velocity v. In formulas (1.1) and (1.2) we let $\nu \to \infty$ ($v \to 0$) and when performing the integration, we find for small velocities of motion the asymptotic expressions:

for the source

$$A_d \sim \frac{\rho Q^2}{16\pi\zeta^2}\left[1 - 4p\sum_{k=1}^{\infty}\frac{k}{(p^2k^2 - 1)^2}\right] \tag{2.1}$$

for the dipole

$$A_d \sim \frac{3\rho m^2}{64\pi\zeta^4}\left[1 - 8p\sum_{k=1}^{\infty}\frac{k(p^2k^2+1)}{(p^2k^2-1)^4}\right] \qquad \left(p = \frac{h}{|\zeta|}\right) \tag{2.2}$$

From (2.1) and (2.2) it follows that for $v \to 0$ the influence of the free surface of the fluid on the quantity A_d is the same as that of a solid wall situated at the level of the stagnant fluid surface.

Now we let $\nu \to 0$ in (1.1) and (1.2). We then obtain for sufficiently large values of the velocity v the following asymptotic expressions:

for the source

$$A_d \sim \frac{\rho Q^2}{16\pi\zeta^2}\left[-1 + 4p\sum_{k=1}^{\infty}(-1)^k\frac{k}{(p^2k^2-1)^2}\right] \tag{2.3}$$

for the dipole

$$A_d \sim \frac{3\rho m^2}{64\pi\zeta^4}\left[-1 + 8p\sum_{k=1}^{\infty}(-1)^k\frac{k(p^2k^2+1)}{(p^2k^2-1)^4}\right] \tag{2.4}$$

For the case $\nu \to 0$ considered here, the field of the velocities induced by the moving singularities asymptotically approaches the field of velocities obtained when the velocity potential is made zero at the free surface of the fluid.

The series of formulas (2.1)'-(2.4) converge quite rapidly ($p > 1$). Fig. 1 shows the curves of the coefficient of the lifting force $a = A_d$: $\rho Q^2/16\pi\zeta^2$ of the source (solid lines) and of the dipole $a = A_d$: $3\rho m^2/64\pi\zeta^4$ (dashed lines) as a function of $p = h/|\zeta|$, constructed on the basis of formulas (2.1)-(2.4). This figure demonstrates the influence of the water depth h on the magnitude of the lifting force of the singularity when the value ζ is fixed for sufficiently small and large velocities of motion. If $p > 4$, then one may assume $a \approx \pm 1$, i.e. the influence is the same as in the case of deep water.

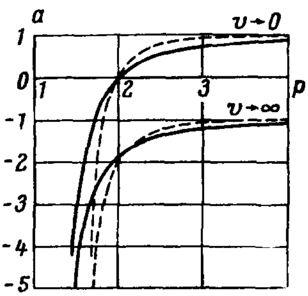

Fig. 1.

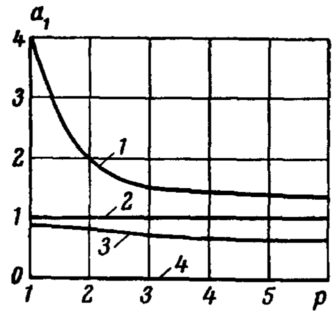

Fig. 2.

3. We will now consider the motion of the submerged source near a vertical wall, situated parallel to the x-axis. We will denote the distance between the source and the vertical wall by b. Consider the direction of the y-axis from the source to the wall to be positive. In accordance with the formulas given in reference [2], we may obtain the following expression for the lateral resistance of the source.

$$R_y = \frac{\rho Q^2}{16\pi b^2}\left\{1 - \frac{1}{(1+q^2)^{3/2}} + \sum_{k=1}^{\infty}(-1)^k\left[\frac{2}{(1+p^2q^2k^2)^{3/2}} - \frac{1}{[1+q^2(pk+1)^2]^{3/2}} - \frac{1}{[1+q^2(pk-1)^2]^{3/2}}\right] + N(h, b\ \zeta, \nu)\right\} \tag{3.1}$$

where

$$N = \frac{8\nu b^2}{\pi}\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} d\theta \int_0^{\infty} \frac{e^{-\lambda h}(1+\operatorname{th}\lambda h)\operatorname{ch}^2\lambda(\zeta+h)}{(\nu\operatorname{th}\lambda h - \lambda\cos^2\theta)\operatorname{ch}\lambda h}\sin(2b\lambda\sin\theta)\sin\theta\,\lambda\,d\lambda \tag{3.2}$$

$q = |\zeta|/b$, and p denote the same parameters as before.

For the case of small velocities of the motion of the source, from equation (3.1) we find

$$R_y \sim \frac{\rho Q^2}{16\pi b^2}\left\{1 + \frac{1}{(1+q^2)^{3/2}} + \sum_{k=1}^{\infty}\left[\frac{2}{(1+p^2q^2k^2)^{3/2}} + \right.\right.$$

$$\left.\left. + \frac{1}{[1+q^2(pk+1)^2]^{3/2}} + \frac{1}{[1+q^2(pk-1)^2]^{3/2}}\right]\right\} \tag{3.3}$$

Hence it is seen that the influence of the free surface of the fluid on the magnitude of R_y is the same as that of the solid wall. For the other limiting case of the velocity of motion of the source, namely ($\nu \to 0$, $v \to \infty$) it is necessary to assume $N = 0$ in (3.1).

Fig. 2 force shows the curves of the coefficient of lateral resistance of the source $a_1 = R_y : \rho Q^2 / 16\pi b^2$ with variation of $p = h|\zeta|$ for certain values of $q = |\zeta| / b$ constructed on the basis of (3.3) and (3.1) for $N = 0$. The following are shown: curve 1 for $v \to 0$, $q = |\zeta| : b = 1$, curve 2 for $\nu \to 0$, $v \to \infty$, $q \to \infty$, curve 3 for $v \to \infty$, $q = 1$, curve 4 for $v \to \infty$, $q \to 0$. For the case of infinite depth of the fluid ($h = \infty$)

$$a_1 = 1 \pm \frac{1}{(1+q^2)^{3/2}} \qquad \begin{matrix}(\text{plus for } v \to 0)\\ (\text{minus for } v \to \infty)\end{matrix} \tag{3.4}$$

The expression (3.3) is also applicable to the determination of the lifting force A_d of the submerged source for sufficiently small values of the velocity in the case of its motion in a canal of infinite depth. In this case b and $|\zeta|$ must exchange their places in formula (3.3) and h denotes the width of the canal (for $h = \infty$ we obtain the value A_d for the case of source motion near one vertical wall). In an exactly similar manner (2.1) allows us to determine the force of the lateral resistance of the source in a stream bounded by two parallel walls.

The motion of the submerged dipole near a vertical wall may be determined similarly.

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